KEY

Chapter 5 Review Packet

1) Simplify to ONE trig function or a number.

a) $\dfrac{\sec^2 x-1}{\sin^2 x}$ $= \dfrac{\tan^2 u}{\sin^2 u} = \dfrac{\frac{\sin^2 u}{\cos^2 u}}{\sin^2 u} = \dfrac{\sin^2 u}{\cos^2 u}\cdot\dfrac{1}{\sin^2 u} = \dfrac{1}{\cos^2 u} = \boxed{\sec^2 u}$

b) $\dfrac{-\sin\left(\frac{\pi}{2}-x\right)}{\cos\left(\frac{\pi}{2}-x\right)}$ $= \dfrac{-\cos u}{\sin u} = \boxed{-\cot u}$

2) Prove the following identities. Be sure to use only ONE side!!

a) $\dfrac{1+\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{1+\sin\theta}=2\sec\theta$ $\quad \dfrac{2}{\cos\theta}$

$\dfrac{(1+\sin\theta)(1+\sin\theta)+\cos^2\theta}{\cos\theta(1+\sin\theta)} = \dfrac{1+2\sin\theta+\overbrace{\sin^2\theta+\cos^2\theta}^{1}}{\cos\theta(1+\sin\theta)} =$

$= \dfrac{2+2\sin\theta}{\cos\theta(1+\sin\theta)} = \dfrac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)} = \boxed{2\sec\theta}$

b) $\cos x-\dfrac{\cos x}{1-\tan x}=\dfrac{\sin x\cos x}{\sin x-\cos x}$

$\dfrac{\cos x(1-\tan x)-\cos x}{1-\tan x} = \dfrac{\cos x-\cos x\tan x-\cos x}{1-\tan x} = \dfrac{-\cos x\tan x}{1-\tan x}$

$= \dfrac{-\cos x\tan x}{1-\frac{\sin x}{\cos x}} = \dfrac{-\cos x\frac{\sin x}{\cos x}}{\frac{\cos x-\sin x}{\cos x}} = -\sin x\cdot\dfrac{\cos x}{\cos x-\sin x}$

$= \dfrac{\sin x\cos x}{-(\cos x-\sin x)} = \boxed{\dfrac{\sin x\cos x}{\sin x-\cos x}}$

Chapter 5 Review Packet

Use the ANY OF THE FORMULAS for the following questions:

Find the **EXACT value** of the expression- this means no decimals!

3) $\sin(75°) = \sin(45+30)$

$= \sin 45 \cos 30 + \cos 45 \sin 30$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

4) $\tan 345° = -\tan(15)$

QIV

$= -\tan \frac{30}{2} = -\frac{1-\cos 30}{\sin 30}$

$= \frac{1-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -(2-\sqrt{3})$

$= \boxed{\sqrt{3}-2}$

5) $\cos 285° = \cos(240+45)$

$= \cos 240 \cos 45 - \sin 240 \sin 45$

$= -\frac{1}{2} \cdot \frac{\sqrt{2}}{2} - -\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$

$= \boxed{\frac{-\sqrt{2}+\sqrt{6}}{4}}$

6) $\sin 105° = \sin(60+45)$

$= \sin 60 \cos 45 + \cos 60 \sin 45$

$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$

$= \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

7) $\cos 165° = \cos(120+45)$

$= \cos 120 \cos 45 - \sin 120 \sin 45$

$= -\frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$

$= \boxed{\frac{-\sqrt{2}-\sqrt{6}}{4}}$

8) $\tan 22.5° = \tan \frac{45}{2}$

$= \frac{1-\cos 45}{\sin 45} = \frac{1-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$

$= \frac{2-\sqrt{2}}{2} \cdot \frac{2}{\sqrt{2}}$

$= \frac{2-\sqrt{2}}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{2\sqrt{2}-2}{2}$

$= \boxed{\sqrt{2}-1}$

Chapter 5 Review Packet

Write the expression as the sine, cosine, or tangent of the angle; you <u>do not</u> have to find the value:

9) $\sin 60° \cos 45° - \cos 60° \sin 45°$

$= \sin(60-45)$

$= \sin 15°$

10) $\cos 45° \cos 120° - \sin 45° \sin 120°$

$= \cos(45+120)$

$= \cos 165°$

11) $\dfrac{\tan 25° + \tan 10°}{1 - \tan 25° \tan 10°}$

$= \tan(25+10)$

$= \tan 35°$

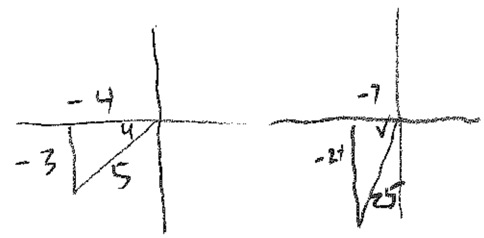
12) $\dfrac{\tan 68° - \tan 115°}{1 + \tan 68° \tan 115°}$

$= \tan(68-115)$

$= \tan(-47°)$

13) Find the **exact value** of the trig function given that

$\sin u = \frac{-3}{5}$ and $\cos v = -\frac{7}{25}$, and where both u and v are in Quadrant III.



$\sin(u-v) = \sin u \cos v - \cos u \sin v$

$= \frac{-3}{5} \cdot \frac{-7}{25} - \frac{-4}{5} \cdot \frac{-24}{25}$

$= \frac{21}{125} - \frac{96}{125} = -\frac{75}{125} = \boxed{-\frac{3}{5}}$

$\cos(u-v) = \cos u \cos v + \sin u \sin v$

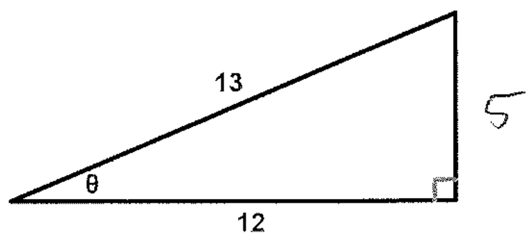
$= \frac{-4}{5} \cdot \frac{-7}{25} + \frac{-3}{5} \cdot \frac{-24}{25}$

$= \frac{28}{125} + \frac{12}{125} = \frac{100}{125} = \boxed{\frac{4}{5}}$

$\tan(u+v) = \dfrac{\tan u + \tan v}{1 - \tan u \tan v} = \dfrac{\frac{3}{4} + \frac{24}{7}}{1 - \frac{3}{4}\left(\frac{24}{7}\right)} = \dfrac{\frac{21+96}{28}}{\frac{28-72}{28}}$

$= \dfrac{117}{28} \cdot \dfrac{28}{-44} = \boxed{-\dfrac{117}{44}}$

14) Use the figure below to find the exact value of the following trig functions:



$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2$

$= \dfrac{144-25}{169} = \boxed{\dfrac{119}{169}}$

$\tan\dfrac{\theta}{2} = \dfrac{1-\cos\theta}{\sin\theta} = \dfrac{1 - \frac{12}{13}}{\frac{5}{13}}$

$= \dfrac{1/13}{5/13} = \dfrac{1}{13} \cdot \dfrac{13}{5} = \boxed{\dfrac{1}{5}}$

$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{5}{13}\right)\left(\frac{12}{13}\right)$

$= \boxed{\dfrac{120}{169}}$

$\sin\dfrac{\theta}{2} = \sqrt{\dfrac{1-\cos\theta}{2}} = \sqrt{\dfrac{1-\frac{12}{13}}{2}}$

$= \sqrt{\dfrac{1/13}{2}} = \sqrt{\dfrac{1}{26}} = \dfrac{1}{\sqrt{26}}$

$= \boxed{\dfrac{\sqrt{26}}{26}}$

Chapter 5 Review Packet

Rewrite the expressions using one of the formulas:

15) $12-24\sin^2 x$ $= 12(1-2\sin^2 x)$

$= \boxed{12\cos 2x}$

16) $\sqrt{\dfrac{1-\cos 6x}{2}} = \sin\dfrac{6x}{2} = \boxed{\sin 3x}$

Solve the following Trig Equations to find the ANGLE(S) in domain $[0,2\pi)$:

17) $\sin^2\theta=\cos^2\theta$

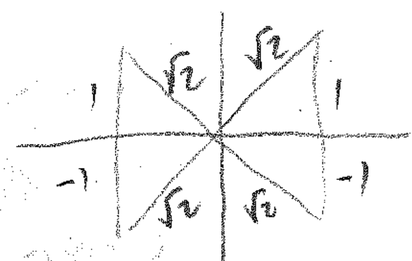
$\sin^2 = 1-\sin^2$

$2\sin^2 = 1$

$\sin^2 = \frac{1}{2}$

$\sin = \pm\frac{1}{\sqrt{2}}$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$



18) $3\sec^2 x-4=0$

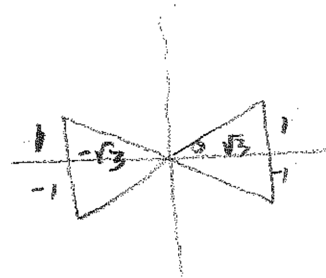
$3(1+\tan^2 x)-4=0$

$3+3\tan^2 x-4=0$

$3\tan^2 x-1=0$

$\tan^2 x=\frac{1}{3}$

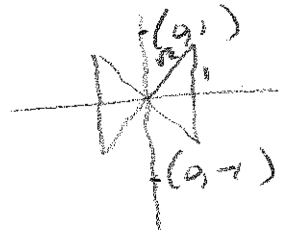
$\tan x=\pm\frac{1}{\sqrt{3}}$



$\boxed{x=\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$

Chapter 5 Review Packet

19) $\sin 2x \sin x = \cos x$



$2\sin x \cos x \sin x - \cos x = 0$

$\cos x(2\sin^2 x - 1) = 0$

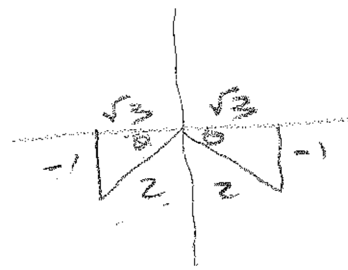
$\cos x = 0$ $\qquad$ $2\sin^2 x - 1 = 0$

$x = \pi/2, 3\pi/2$ $\qquad$ $\sin^2 x = \frac{1}{2}$

$\sin x = \pm \frac{1}{\sqrt{2}} \Rightarrow x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

18) $\sin 2x + \cos x = 0$

$2\sin x \cos x + \cos x = 0$



$\cos x(2\sin x + 1) = 0$

$\cos x = 0$ $\qquad$ $2\sin x + 1 = 0$

$x = \pi/2, 3\pi/2$ $\qquad$ $\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$